CHAPTER

3

MATRICES

CONCEPTS AND FORMULAE

1. A matrix is an ordered rectangular array of numbers or functions. The number or functions are called the elements or the entries of the matrix. It is denoted by the symbol [] or ().
2. **Order of matrix:** The size or order of a matrix is established by looking first at the number of rows and second at the number of columns. The order is written as $m \times n$, where m represents number of rows and n represents number of columns.

   In general, $m \times n$ matrix has the following rectangular array:

$$A_{mn} = \begin{bmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1m} \\ a_{21} & a_{22} & a_{23} & \cdots & a_{2m} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ a_{m1} & a_{m2} & a_{m3} & \cdots & a_{mn} \end{bmatrix} m \times n$$

   or $[a_{ij}]\ m \times n$, where $1 \le i \le m,\ 1 \le j \le n;\ i, j \in N$.
3. When number of rows in a matrix equals number of column in it, the matrix is called a square matrix.
4. A matrix having one row and any number of columns is called a row matrix. In other words, matrix of order $m \times n$ is always a column matrix.
5. A matrix is said to be a zero matrix or null matrix if are its element are zeroes.
6. A square matrix $[a_{ij}]$ is said to be a diagonal matrix if $a_{ij} = 0$ for $i \neq j$.
7. A square matrix having the elements in main diagonal as 1 and rest are zeroes is called an identity or unit matrix.
8. **A square matrix whose** all non-diagonal elements are zeroes and diagonal elements are equal, is called a scalar matrix. In other words, if $A = [a_{ij}]$ is a square matrix and $a_{ij} = \begin{cases} \alpha & i = j \\ 0 & i \neq j \end{cases}$ then A is a scalar matrix.
9. Two matrices A and B are said to be equal, if
   (*i*) order of A and B are same.
   (*ii*) **Corresponding elements** of A and B are same i.e., $a_{ij} = b_{ij}$
10. If A is an $m \times n$ matrix and k is a scalar, then

$$kA = k\,[a_{ij}]\ m \times n = [k\,(a_{ij})]\ m \times n.$$

11. **Addition and subtraction of Two matrices:** Addition and subtraction of two matrices is defined if order of both the matrices are same.

   – If

$$A = [a_{ij}]\ m \times n \text{ and } B = [b_{ij}]\ m \times n \text{ then}$$
$$A + B = [a_{ij} + b_{ij}]\ m \times n\ 1 \le i \le m,\ 1 \le j \le m$$

   – If

$$A = [a_{ij}]\ m \times n \text{ and } B = [b_{ij}]\ m \times n \text{ then}$$
$$A + B = [a_{ij} + b_{ij}]\ m \times n\ 1 \le i \le m,\ 1 \le j < m$$

Properties of Matrix Addition:

Let $A = [a_{ij}]$ $B = [b_{ij}]$ and $C = [c_{ij}]$ are three matrices of same order.

(*i*) Commutative law: If A $[a_{ij}]$ and $B = [b_{ij}]$ are matrices of the same order, say $m \times n$, then

$$A + B = B + A$$

(*ii*) Associative law: Matrix addition is associative

$$A + (B + C) = (A + B) + C$$

(*iii*) **Existence of additive identity:** Let $A = [a_{ij}]$ $m \times n$ matrix and O be $m \times n$ zero matrix then

$$A + O = O + A = A$$

(*iv*) **Existence of additive inverse:** Let A be a matrix, then (–A) is the additive inverse of A.

$$A + (-A) = O = (-A) + A$$

(*v*) **Distributive law:** If $A = [a_{ij}]$ and $B = [b_{ij}]$ and $C = [c_{ij}]$ be three matrices of same order, say $m \times n$, then

$$A(B + C) = AB + AC.$$

12. Multiplication of a matrix by scalar number: Let $A = [a_{ij}]$ $m \times n$ be a matrix and k is a scalar then kA is another matrix obtained by multiplying each element of A by the scalar k.

If $\quad A = [a_{ij}]\ m \times n$ then $kA = [k\,a_{ij}]\ m \times n$.

Properties of scalar multiplication:

If A and B are two matrices of the same order and k, l are scalars then

(*i*) $k(A + B) = kA + kB$

(*ii*) $(k + l)A = kA + lA$

(*iii*) $(kl)A = k(lA) = l(kA)$

(*iv*) $(-k)A = -(kA) = k(-A)$

(*v*) $1(A) = A$

(*vi*) $(-1)A = -A$

13. Multiplication of Two Matrices: Let A and B be two matrices. Then their product AB is defined, if the number of columns in matrix A is equal to the number of rows in matrix B.

Let $\quad A = [a_{ij}]\ m \times n$ and $B = [b_{ij}]\ n \times p$ then

$$AB = [c_{ik}]\ m \times p \text{ where } c_{ik} = \sum_{j=1}^{n} a_{ij}\, b_{jk}$$

If the i^{th} row of A is $[a_{i1}\ a_{i2}\ ....\ a_{in}]$ and the k^{th} column of B is $\begin{bmatrix} b_{1k} \\ b_{2k} \\ \vdots \\ b_{nk} \end{bmatrix}$ then

$$c_{ik} = a_{i1}\, b_{1k} + a_{i2}\, b_{2k} + ... + a_{in}\, b_{nk} = \sum_{j=1}^{n} a_{ij}\, b_{jk}$$

14. Matrix multiplication has the following properties:

(*i*) Matrix multiplication is not commutative.

(*ii*) Matrix multiplication is associative i.e. (AB)C = A(BC)

(*iii*) Matrix multiplication is distributive over matrix addition.

$$A(B + C) = AB + AC \text{ and } (B + C)A = BA + CA$$

(*iv*) If A is an $m \times n$ matrix, then $I_m A = A = AI_n$

(*v*) If A is an $m \times n$ matrix and O is a null matrix, then $A_{m \times n} O_{n \times p} = O_{m \times p}$ and $O_{p \times m} \times A_{m \times n} = O_{p \times n}$

The product of a matrix with a null matrix is a null matrix.

15. If A is a square matrix then we define,

$A' = A$ and $A^{n+1} = A^n.A$.

16. Any square matrix A can be expressed as the sum of a symmetric and skew symmetric matrix.

$$A = \frac{A + A'}{2} + \frac{A - A'}{2}.$$

EXERCISE 3.1

Multiple Choice Questions (MCQs)

1. The order of matrix $\begin{bmatrix} 2 & 4 & 3 \\ 5 & -1 & -2 \end{bmatrix}$ is

(a) 2×2 (b) 2×3 (c) 3×2 (d) 3×3

2. If a matrix has 7 elements, the possible order it can have are

(a) 1×7 or 7×1 (b) 1×1 or 7×7 (c) $2 \times 3, 3 \times 2$ (d) None of these

3. A 3×3 matrix $A = [a_{ij}]$ whose elements are given by $a_{ij} = (i + j)^2$ is

(a) $\begin{bmatrix} 4 & 9 & 16 \\ 9 & 16 & 25 \\ 16 & 25 & 18 \end{bmatrix}$ (b) $\begin{bmatrix} 4 & 9 & 16 \\ 9 & 16 & 25 \\ 6 & 25 & 26 \end{bmatrix}$ (c) $\begin{bmatrix} 4 & 9 & 16 \\ 9 & 16 & 25 \\ 16 & 25 & 36 \end{bmatrix}$ (d) None of these

Very Short Answer Type Questions (1 Mark)

4. If a matrix has 8 elements, what are the possible orders it can have? What if it has 5 elements?

5. Construct a 2×2 matrix $A = [a_{ij}]$ whose elements are given by

(i) $a_{ij} = \dfrac{i}{j}$ [*CBSE* 2008 *C*, 2011]

(ii) $a_{ij} = i + 2j$ [*CBSE* 2008]

6. A matrix has 6 elements. What are the possible orders it can have? What if it has 3 elements?

Short Answer Type Questions (2 Marks)

7. Find the trace of the matrix $\begin{bmatrix} 2 & 5 & 9 \\ 7 & -5 & 3 \\ 2 & 6 & 8 \end{bmatrix}$

8. Find the value of x, y, z and a so that $\begin{bmatrix} x-3 & 2y-8 \\ z+2 & 6 \end{bmatrix} = \begin{bmatrix} 5 & 2 \\ -2 & a-4 \end{bmatrix}$

9. If $3A - B = \begin{bmatrix} 5 & 0 \\ 1 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 4 & 3 \\ 2 & 5 \end{bmatrix}$, then find the matrix A. [CBSE 2012 C]

Long Answer Type Questions—I (4 Marks)

10. (*i*) If $2\begin{bmatrix} 3 & 4 \\ 5 & x \end{bmatrix} + \begin{bmatrix} 1 & y \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 7 & 0 \\ 10 & 5 \end{bmatrix}$, find $(x - y)$. [CBSE 2014]

(*ii*) Find x, y, z, w if $\begin{bmatrix} x & 6 \\ -2 & 2w \end{bmatrix} + \begin{bmatrix} 4 & x+y \\ x+w & 3 \end{bmatrix} = 3\begin{bmatrix} x & y \\ z & w \end{bmatrix}$ [*Panjab Board* (2005)]

11. If $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix}$ and $B = \begin{bmatrix} -2 & -2 \\ 1 & -5 \\ 4 & 3 \end{bmatrix}$ then find $X = \begin{bmatrix} p & q \\ r & s \\ t & u \end{bmatrix}$ so that $A + B - X = 0$.

12. Find x, y, z and w if $3\begin{bmatrix} x & y \\ z & w \end{bmatrix} = \begin{bmatrix} x & 6 \\ -1 & 2w \end{bmatrix} + \begin{bmatrix} 4 & x+y \\ 1+w & 3 \end{bmatrix}$

Long Answer Type Questions—II (6 Marks)

13. If $A = \text{diag}\,[2 \quad 9 \quad 4]$, $B = \text{diag}\,[-3 \quad 7 \quad 6]$, find (*i*) $A - B$ (*ii*) $9A - 11B$.

14. (*i*) Find X and Y if $X + Y = \begin{bmatrix} 7 & 0 \\ 2 & 5 \end{bmatrix}$ and $X - Y = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$

(*ii*) Find X and Y, given that $3X - Y = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$ and $X - 3Y = \begin{bmatrix} 0 & -1 \\ 0 & -1 \end{bmatrix}$

[CBSE 2002]

15. Find X and Y if $2X + Y = \begin{bmatrix} 4 & 4 & 7 \\ 7 & 3 & 4 \end{bmatrix}$ and $X - 2Y = \begin{bmatrix} -3 & 2 & 1 \\ 1 & -1 & 2 \end{bmatrix}$.

Answers